BOUNDARY-LAYER INTERACTION WITH A NONEQUILIBRIUM TWO-PHASE STREAM ON A SURFACE BEING BURNED OUT IN AN AXISYMMETRIC LAVAL NOZZLE

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§1. The analysis of a two-fluid two-dimensional stream is based on the Rakhmatulin model of a two-velocity and two-temperature continuous medium in which the real flow is replaced by a mutually penetrating flow of two interacting continuous media [1-8].

The initial stationary problem for the analysis of the flow for a given nozzle contour is replaced by a nonstationary problem, i.e., a buildup process is used to obtain a stationary flow picture. The nonstationary sub- and supersonic flow equations are equations of hyperbolic type. As a result of neglecting the particle volume, the stationary equations of particle motion are also of hyperbolic type.

The system of equations has the form [2, 4, 6]

$$\frac{\partial}{\partial t}(y\rho)+\frac{\partial}{\partial x}(ym)+\frac{\partial}{\partial y}(yn)=0; \tag{1.1}$$
$$\frac{\partial}{\partial t}(ym)+\frac{\partial}{\partial x}\left[y\left(\frac{m^2}{\rho}+p\right)\right]+\frac{\partial}{\partial y}\left(y\frac{mn}{\rho}\right)=yC_R\left(m_p-\frac{\rho_p}{\rho}m\right);$$
$$\frac{\partial}{\partial t}(yn)+\frac{\partial}{\partial x}\left(\frac{ymn}{\rho}\right)+\frac{\partial}{\partial y}\left[y\left(\frac{n^2}{\rho}+p\right)\right]=p+yC_R\left(n_p-\frac{\rho_p}{\rho}n\right);$$
$$\frac{\partial}{\partial t}(yE)+\frac{\partial}{\partial x}\left[\frac{ym}{\rho}(E+p)\right]+\frac{\partial}{\partial y}\left[\frac{yn}{\rho}(E+p)\right]=y\left\{C_\alpha\left(\rho_pT_p-\frac{\rho_p}{\rho}p\right)\right.$$
$$\left.+C_R\left[m_p(u_p-u)+n_p(v_p-v)\right]\right\};$$
$$p=\frac{k-1}{k}\left(\rho H-\frac{m^2+n^2}{2\rho}\right);\quad m=\rho u;\quad n=\rho v;$$

and for the particle "gas"

$$\frac{\partial}{\partial x}(ym_p)+\frac{\partial}{\partial y}(yn_p)=0; \tag{1.2}$$
$$\frac{\partial}{\partial x}\left(\frac{ym_p^2}{\rho_p}\right)+\frac{\partial}{\partial y}\left(\frac{ym_pn_p}{\rho_p}\right)=yC_R\left(\frac{m\rho_p}{\rho}-m_p\right);$$
$$\frac{\partial}{\partial x}\left(\frac{ym_pn_p}{\rho_p}\right)+\frac{\partial}{\partial y}\left(\frac{yn_p^2}{\rho_p}\right)=yC_R\left(\frac{n\rho_p}{\rho}-n_p\right);$$
$$\frac{\partial}{\partial x}\left(\frac{yQ_pm_p}{\rho_p}\right)+\frac{\partial}{\partial y}\left(\frac{yQ_pn_p}{\rho_p}\right)=y\frac{C_\alpha}{C}\rho_p(T-T_p);$$
$$Q_p=\rho_pT_p;\ m_p=\rho_pu_p;\ n_p=\rho_pv_p.$$

All the quantities in (1.1) and (1.2) are dimensionless: The linear dimensions are referred to the radius of the nozzle critical section r_*, the velocity projections of the gas u^0 and v^0 of the particles with subscript p are referred to the speed of sound of the gas a_*; the magnitudes of the density ρ^0, pressure p^0, temperature T^0, enthalpy H^0, and energy E^0 are referred to the corresponding quantities at the stagnation temperature T_0 in the forechamber; the time t^0 is referred to the quantity r_*/a_*, and the specific heat c_p^0 is referred to the gas constant R.

It is assumed that all the parameters across the initial section of the nozzle are distributed uniformly and are obtained from a computation of the one-dimensional equilibrium two-phase stream without particle lag.

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Moreover, homogeneous distributions of the enthalpy H and entropy S are assumed, and the transverse velocity component v varies linearly along the radius for the gas. The density ρ_p is constant for the particles and depends on the relative weight concentration z:

for $x = x_0$, $0 \leq y \leq \delta(x_0)$

$$S = C_1, H = C_2, n = m \frac{y}{\delta(x_0)} \frac{d}{dx} \delta(x_0) \Big|_{x=x_0},$$

$$u_p = u, \quad v_p = v, \quad T_p = T, \quad \rho_p = \rho \frac{z}{1-z},$$

where $\delta(x)$ is the equation of the nozzle contour.

The nonpenetration condition is posed for the gas on the nozzle wall:

$$\text{for } y = \delta(x) \text{ and } x_0 \leqslant x \leqslant x_c, \quad n = m \frac{d}{dx} \delta(x).$$

No reflection is assumed for the particles at the wall, and no additional conditions are imposed.

On the axis of symmetry

$$\text{for } y = 0 \text{ and } x_0 \leqslant x \leqslant x_c,$$

$$\partial m/\partial y = \partial m_p/\partial y = \partial p/\partial y = \partial E/\partial y = \partial T_p/\partial y = \partial \rho_p/\partial y = 0, \ n = 0, \ n_p = 0.$$

A region of pure gas appears near the wall because of particles lagging behind the gas. The particle limit line was sought by means of the equation $d\Gamma\ dx = n_p/m_p$ in the domain $0 \leq y \leq \Gamma(x)$, $x_0 \leq x \leq x_c$.

The buildup method for the gas equation becomes superfluous for the supersonic region of the nozzle, since the stationary equations are of hyperbolic type. The stationary systems of equations for the gas and particles (1.1), (1.2) were solved in this region. The solutions for $x = x_c$, obtained from an analysis in the supersonic domain, are the input parameters for these equations.

To solve the systems of equations presented above, the independent variables are transformed. In the system (1.1)

$$t = t, \ x = x, \ \xi = y/\delta(x),$$

and in Eqs. (1.2)

$$t = t, \ x = x, \ \psi = y/\Gamma(x).$$

This transformation excludes the appearance of nonregular nodes reducing the accuracy of the computation in a solution using a finite-difference scheme.

The transformed systems of equations were written in divergent form. Their numerical solution is based on the MacCormack two-step difference scheme [9, 10], which is a predictor–corrector scheme of second-order accuracy in space and time on smooth solutions. This scheme permits a thorough computation of singularities of shock wave and rarefaction wave type if the initial differential equations are written in divergent form.

If the transformed systems of equations are written in vector form

$$\frac{\partial \mathbf{f}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial \xi} + \mathbf{H} = 0,$$

where **f**, **F**, **G**, and **H** are vectors with components dependent on the parameters being computed, then the two-step MacCormack difference schemes can be represented in the following form:

for a nonstationary system

$$\mathbf{f}_{i,j}^{(n+1)^\circ} = \mathbf{f}_{i,j}^n - \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i,j}^n - \mathbf{F}_{i-1,j}^n\right) - \frac{\Delta t}{\Delta \xi}\left(\mathbf{G}_{i,j}^n - \mathbf{G}_{i,j-1}^n\right) - \Delta t \cdot \mathbf{H}_{i,j}^n;$$

$$\mathbf{f}_{i,j}^{n+1} = \frac{1}{2}\left[\mathbf{f}_{i,j}^n + \mathbf{f}_{i,j}^{(n+1)^\circ} - \frac{\Delta t}{\Delta x}\left(\mathbf{F}_{i+1,j}^{(n+1)^\circ} - \mathbf{F}_{i,j}^{(n+1)^\circ}\right) - \frac{\Delta t}{\Delta \xi}\left(\mathbf{G}_{i,j+1}^{(n+1)^\circ} - \mathbf{G}_{i,j}^{(n+1)^\circ}\right) - \Delta t \cdot \mathbf{H}_{i,j}^{(n+1)^\circ}\right];$$

for a stationary system

$$\mathbf{F}_{(i+1)^\circ,j} = \mathbf{F}_{i,j} - \frac{\Delta x}{\Delta \psi}\left(\mathbf{G}_{i,j+1} - \mathbf{G}_{i,j}\right) - \Delta x \cdot \mathbf{H}_{i,j};$$

$$\mathbf{F}_{i+1,j} = \frac{1}{2}\left[\mathbf{F}_{i,j} + \mathbf{F}_{(i+1)^\circ,j} - \frac{\Delta x}{\Delta \psi}\left(\mathbf{G}_{(i+1)^\circ,j} - \mathbf{G}_{(i+1)^\circ,j-1}\right) - \Delta x \cdot \mathbf{H}_{(i+1)^\circ,j}\right];$$

for the limit line equation

$$\Gamma_{(i+1)^\circ} = \Gamma_i + \Delta x \left(\frac{n_p}{m_p}\right)_i;$$

$$\Gamma_{i+1} = \frac{1}{2}\left[\Gamma_i + \Gamma_{(i+1)^\circ} + \Delta x \left(\frac{n_p}{m_p}\right)_{(i+1)^\circ}\right].$$

§2. Boundary-layer growth is considered to take account of the viscous interaction with the wall during two-phase nonequilibrium stream flow in a nozzle. A turbulent boundary layer (TBL) is assumed on the nozzle wall. The boundary-layer characteristics are determined on the burned up (entrainable and displaceable) rough multilayered nonstationarily heated surface around which flows the two-phase nonequilibrium multicomponent gas mixture which goes into chemical reaction with the wall material. The analysis is performed using the method of [11] by solving the integral equations of the momenta, the energy, the conservation of the components in the reacting mixtures, and the nonstationary heat-conduction equation across the axisymmetric channel walls. The approach proposed in [12] was used to close the turbulent boundary-layer equations.

The method of computing the turbulent boundary-layer characteristics [11] permits taking account of the nonstationarity of the heat elimination in the wall, the arbitrary distribution law of the wall temperature along the stream and in time, the step entrainment of the wall material along the length of the nozzle profile, and the roughness of the material of the streamlined surface.

The method of computing the quasistationary TBL is based on the following assumptions.

1. The turbulent boundary layer starts from the initial channel section being computed.

2. The nonstationarity of the process is taken into account by solving the heat-conduction equation at the wall by finite differences. The wall temperature T_W determined from the solution in each time interval Δt is the boundary condition for a computation of the TBL parameters in the quasistationary approach.

3. Quantities on the wall, obtained from a computation of the two-phase two-dimensional nonequilibrium stream, are parameters on the outer boundary-layer limit.

4. The influence of the condensed phase on the heat- and mass-transfer and friction processes in the boundary layer is taken into account in terms of the average values of the thermal conductivity, specific heat, enthalpy, molecular weight, and the nonisothermy and blowing parameter, respectively.

5. Entrainment of the erosion-resistant materials (ERM) and the heat shields (HS), consisting of ablating materials on the basis of macromolecular compounds [13, 14], occurs because of their thermal decomposition and the chemical erosion of the coke residue [15].

6. The Prandtl Pr, Schmidt Sc, and Lewis Le numbers equal one. Dissimilarity of the friction and heat- and mass-transfer processes is taken into account by means of the integral relations for the boundary layer and the friction and heat-exchange laws.

7. Roughness of the material surface is due to its ablation. The roughness is assumed sandy in its structure, corresponding to a class of coatings [16].

The integral equations of the momenta, energy, and conservation of the components for the boundary layer in an axisymmetric burning-up (with a transverse stream of substance) channel are written in the form

$$\frac{d\mathrm{Re}^{**}}{dx} + \frac{\mathrm{Re}^{**}}{u_0^0}\frac{du_0^0}{dx}(1+H) + \frac{\mathrm{Re}^{**}}{r^0}\frac{dr^0}{dx} = \mathrm{Re}_L\frac{c_f}{2}(1+b_1), \tag{2.1}$$

$$\frac{1}{r^0\Delta h^0}\frac{d}{dx}\left(\mathrm{Re}_T^{**}r^0\Delta h^0\right) = \mathrm{Re}_L\mathrm{St}(1+b_1),$$

$$\frac{1}{r^0\Delta k^0}\frac{d}{dx}\left(\mathrm{Re}_g^{**}r^0\Delta k^0\right) = \mathrm{Re}_L\mathrm{St}_g(1+b_1),$$

where the differences between the enthalpies Δh^0 and the reduced concentrations Δk^0 [15] are taken over the boundary-layer thickness $\Delta h^0 = h_W^0 - h_W^{0*}$, $\Delta k^0 = k_0^0 - k_W^0$; the penetrability parameter is $b_1 = 2j_W/\rho_0 u_0 c_f$; r^0 is the running radius of the channel; and the friction c_f and heat- and mass-transfer (St, St_g) coefficients are defined in terms of the relative quantity Ψ

$$c_f = c_{f0}\Psi,\ \mathrm{St} = \mathrm{St}_0\Psi,\ \mathrm{St}_g = \mathrm{St}_{0g}\Psi.$$

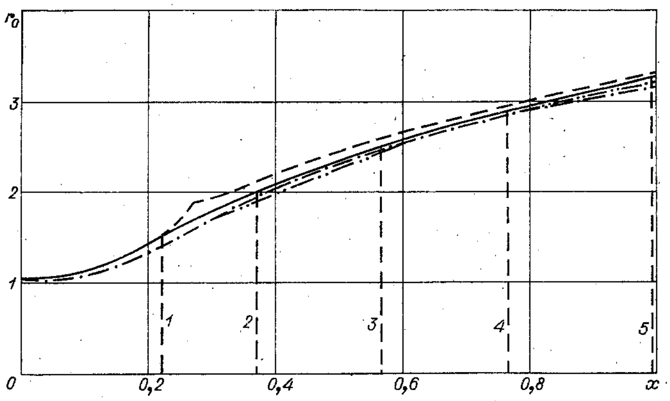

Fig. 1

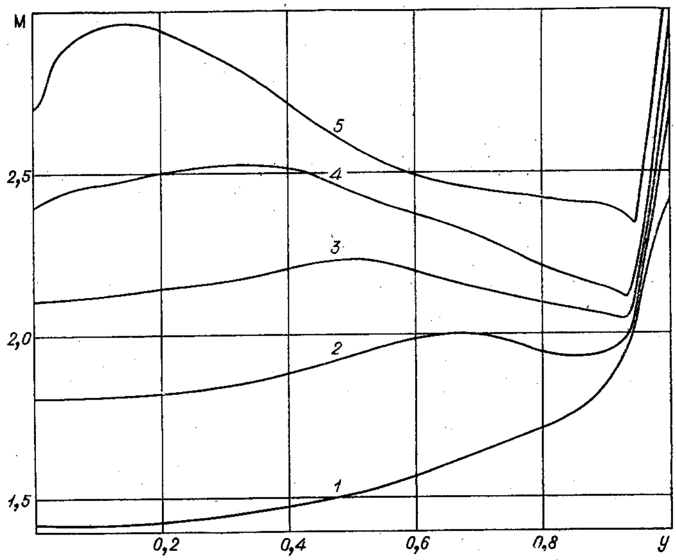

Fig. 2

The friction laws for standard conditions have the form

$$c_{f0} = \frac{0.0256}{\mathrm{Re}^{**0.25}}\left(\frac{\mu_w}{\mu_{00}}\right)^{0.25} \text{ for } \quad \mathrm{Re}^{**} \leqslant 3 \cdot 10^3,$$

$$c_{f0} = \frac{0.0131}{\mathrm{Re}^{**1/6}}\left(\frac{\mu_w}{\mu_{00}}\right)^{0.25} \text{ for } \quad \mathrm{Re}^{**} > 3 \cdot 10^3.$$

The heat- and mass-transfer laws are

$$\mathrm{St}_0 = \frac{0.0128}{\mathrm{Re}_T^{**0.25}\mathrm{Pr}^{0.75}}\left(\frac{\mu_w}{\mu_{00}}\right)^{0.25}; \quad \mathrm{St}_{0g} = \frac{0.0128}{\mathrm{Re}_g^{**}\mathrm{Sc}^{0.75}}\left(\frac{\mu_w}{\mu_{00}}\right)^{0.25},$$

where Re^{**}, Re_T, Re_g^{**} are the Reynolds numbers over the thicknesses of the loss of momentum, energy, and diffusion; μ is the viscosity of the two-phase mixture, which depends on the temperature according to the Sutherland formula; and the subscript w refers to parameters at the wall and the subscript 00, to stagnation parameters.

The relative friction law Ψ for conditions of taking account of the factors listed is used in the form of the following approximate dependence:

$$\Psi = \left(\frac{2}{\sqrt{\psi_1}+1}\right)^2 \left(\frac{\sqrt{\psi_3}+1}{\sqrt{\psi_2}+1}\right)^2 \left(1 - \frac{b_1\Psi}{b_{\mathrm{cr}}}\right)^2 \frac{\left(\operatorname{arctg}\sqrt{r\frac{k-1}{2}\mathrm{M}^2}\right)^2}{r\frac{k-1}{2}\mathrm{M}^2} \left(\frac{\varkappa C + \ln \delta_s^+}{\varkappa C + \ln \delta_r^+ - \varkappa\Phi}\right)^2. \tag{2.2}$$

The critical penetrability parameter is

$$b_{\mathrm{cr}} = (b_{\mathrm{cr}_1} \cdot b_{\mathrm{cr}_2}/b_{\mathrm{cr}_3})\Psi_{\mathrm{M}}^{1.125}\,[(1+\psi_3)/(1+\psi_1)(1+\psi_2)]^{0.05}. \tag{2.3}$$

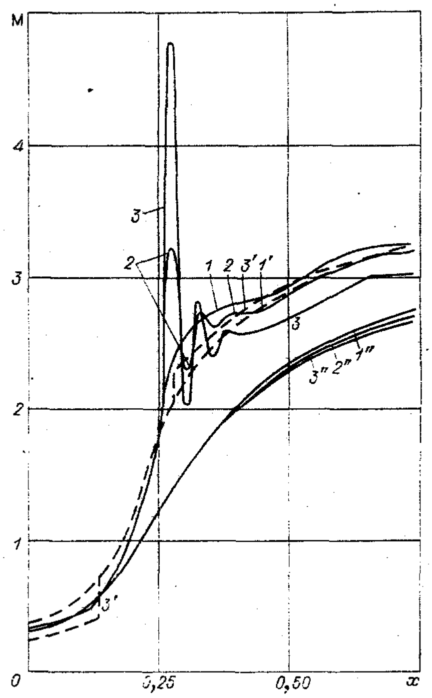

Fig. 3

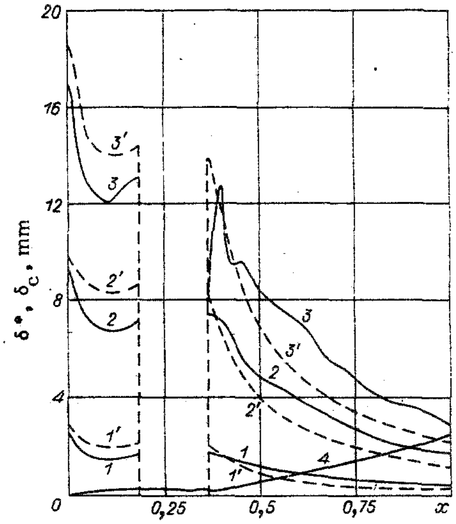

Fig. 4

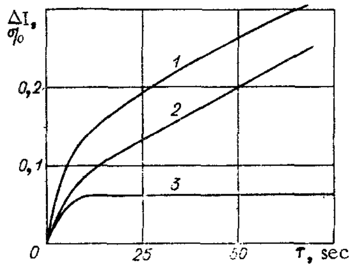

Fig. 5

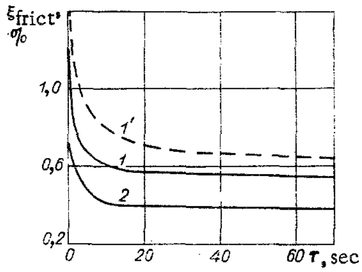

Fig. 6

Formulas (2.2) and (2.3) approximate the results of a numerical computation to 3% accuracy for an arbitrary change in the parameters

$$\psi_1 = h_W^0 / h_W^{0*},\quad \psi_2 = m_0/m_W,\quad \psi_3 = c_{pW}/c_{p0},\quad b_1$$

and the Mach number M. Evaluation of b_{cr_i} is performed according to [17].

Each factor of (2.2) takes account of the respective influence of the following factors: the nonisothermy ψ_1; inhomogeneity of blowing ψ_2, ψ_3; the transverse stream of substance because of thermal decomposition b_G and entrainment of its coke residue b_C; influence of compressibility (M is the Mach number); and roughness. But a simple multiplication of the influence of the perturbing factors noted by the relative friction coefficient does not denote their additive effect; interference occurs here.

For example, the generalized nonisothermy parameter ψ_1 is defined in terms of the ratio between the enthalpy of the gas mixture on the wall h_W^0 and the stagnation enthalpy h_W^{0*}, where h_W^0 depends on the composition and quantity of gas blown into the boundary layer $b_1 = b_G + b_C$ because of thermal decomposition of the wall material and erosion of the coke residue, as well as from the thermal effects of the chemical reactions occurring on both the wall surface and within the material during its pyrolysis [11]. The blowing effect is itself taken into account by the third member in (2.2). The second member of this equation again takes account of the inhomogeneity of the gas blown into the boundary layer as compared with the stream on the outer boundary-layer limit. The influence of roughness is manifest in terms of the height of the protuberances [the last member in (2.2)], but, on the other hand, this influence varies as the nonisothermy, blowing, and compressibility parameters change. The description of the mutual influence under the combined effect of the different factors could be continued.

The last member in (2.2) takes account of the influence of roughness on the friction coefficient $\Psi_r = c_{fr}/c_f$. The surface roughness is given in terms of the characteristic Reynolds number $Re_k = \rho_0 u_0 k/\mu_0$, written over the height of the protuberances k. The parameters in this factor are calculated as follows:

$$k^+ = \frac{ku_*}{\nu_w} = Re_k \frac{\mu_0}{\mu_w} \sqrt{\frac{c_{fr}}{2}\frac{\rho_w}{\rho_0}};$$
$$\delta^+ = \frac{\delta u_*}{\nu_w} = Re^{**}\frac{\delta}{\delta^{**}} \sqrt{\frac{c_f}{2}\frac{\rho_w}{\rho_0}},$$
$$\frac{\delta^{**}}{\delta} = \frac{1}{\varkappa}\sqrt{\frac{c_f}{2}} - \frac{c_f}{2\varkappa^2}\left(2 - \frac{\alpha}{2} + \beta\right), \quad (2.4)$$

where $\varkappa = 0.4$, $C = 5.5$, $\alpha = 1 - h_w^*/h_w$, $\beta = r\frac{k-1}{2}M^2\frac{T_0}{T_w}$, u_* is the dynamic velocity, and ν_w is the kinematic viscosity in (2.2) and (2.4).

The function Ψ_r is determined by successive approximations, where the surface is assumed smooth $\Psi_r = 1$ as the first approximation, $c_f = c_{fs}$ is taken for the smooth surface, and $c_f = c_{fr}$ for a rough surface with identical number Re^{**}. This refers to the determination of the quantities δ^+, δ, δ^{**}.

A nonstationary heat-conduction equation (without taking account of the spreading along the wall)

$$c\gamma\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial r}\left(\lambda\frac{\partial T}{\partial r}\right) + \frac{1}{r}\lambda\frac{\partial T}{\partial r}, \quad (2.5)$$

is solved to take account of the nonstationary heat elimination in the multilayered cylindrical wall. The finite-difference scheme of this equation for variables of the temperature of the thermophysical properties of the materials is written in [11]. A condition of the third kind borrowed from [18] is the boundary condition on the unentrainable wall and on the entrainable wall up to the beginning of the erosion. After the beginning of entrainment, a constant entrainment temperature T_w [11], determined with the heat of reaction of thermal decomposition of the material in a narrow front, the heat of chemical reactions on the boundary with the two-phase stream in the diffusion reaction mode, and also the thermophysical properties of the materials taken into account, is set for the entrainable materials. The initial temperature distribution over the wall thickness is given. The conditions of equal temperatures and heat fluxes are imposed at the junction of two materials.

The magnitudes of the thicknesses of entrainment and of the coke layer, the heat fluxes at the wall, the loss of specific impulse by friction, the thermal state of the wall, etc., in time are determined by the alternate solution of (2.5) with (2.1) when using the parameters on the outer boundary-layer limit, which were obtained from the system (1.1), (1.2) for the time interval in which the process is considered nonstationary. The parameters of the two-phase flow with the change in profile (TBL characteristics) are finally refined by successive approximations, and then the passage to a new time interval is realized.

§3. Computations were performed by the method elucidated for the nozzle contour shown in Fig. 1 for the following mixture parameters in the prechamber: $p_{00} = 43$ atm, $T_{00} = 3450°K$, weight fraction of particles $z = 0.32$, $k = 1.19$, and particle diameter $d_p = 2\ \mu$. The initial nozzle profile is shown by the solid line, and the final profile, distorted because of entrainment of the wall material, is shown by the dashed line; while the dashed-dot lines are the limit lines for the particles in the initial and final profiles. The Mach number distribution across the nozzle for the different sections 1-5 (see Fig. 1) in the supersonic part is shown in Fig. 2. A high-velocity and low-temperature pure gas region, extracted well by this computational scheme, appears near the wall. The particle velocity is less than the gas velocity, and the maximum lag is observed in the domain of highest velocity gradients, i.e., in the supersonic domain.

The Mach number distribution along the nozzle length is shown in Fig. 3. The magnitudes on the nozzle wall and axis, respectively, are noted by lines 1 and 1" for the initial profile (without erosion) by means of a computation for a two-phase, two-dimensional nonequilibrium stream, while curve 1' is for an equilibrium mixture according to a one-dimensional analysis. As the nozzle profile is distorted because of entrainment of the wall material (t^0 varies from 0 to 70 sec), the gasdynamic stream parameters at the wall vary strongly for a two-dimensional, two-phase problem (curves 2 and 3) at the site of the appearance of a step on the juncture between the unentrainable and entrainable materials. Initially rarefaction, then a compression shock, and later interference during interaction with the stream core occur in the region of sudden nozzle profile expansion because of the erosion. The intensity of local fluctuations in the gasdynamic parameters grows with the increase in the erosion (curve 2 at $t^0 = 30$ sec and curve 3 at $t^0 = 70$ sec). These parameter changes do not reach the stream axis in practice (curves 1"-3" at the appropriate times). Changes in the Mach numbers for a one-dimensional equilibrium computation of the initial profile (curve 1') and one distorted because of entrainment (curve 3' at $t^0=70$ sec) are represented here.

The change in the gasdynamic parameters results in a corresponding change in the quantity of the entrainment along the nozzle length, represented in Fig. 4, where curves 1-3 characterize linear erosion of the nozzle profile at the times 10, 34, and 70 sec with the flow of the two-dimensional two-phase nonequilibrium mixture taken into account, while the corresponding dashed curves are for one-dimensional equilibrium gasdynamics. This is related to the change in stream parameters on the wall (see Fig. 3). It must be noted that these peculiarities of wall material entrainment with the two-dimensional and two-phase gasdynamics taken into account duplicate the results of experiments well.

A change in the boundary-layer displacement thickness, which varies slightly because of nozzle wall heating and entrainment, is shown by curve 4 in Fig. 4. It is seen that the change in the initial profile to the nozzle exit is significant because of the boundary-layer displacement thickness and is commensurate with the thickness of the entrained material (this must be taken into account).

Additional losses in the nozzle specific impulse ΔI, shown by curve 1 in Fig. 5 for the entrainable nozzle with the rough wall, by curve 2 for an entrainable nozzle with a smooth wall, and by curve 3 for an unentrainable nozzle with a smooth wall, appear because of nozzle profile distortion (displacement and entrainment thicknesses). It is seen that ΔI grows as the thickness of the entrained material increases (curves 1 and 2). Roughness introduces an additional quantity because of displacement thickness. The displacement thickness on the smooth wall without entrainment for this profile results in negligible additional losses in the specific impulse, commensurate with the accuracy of the computations. Losses of the specific impulse due to friction ξ_{frict} with (and without) the two-dimensionality and nonequilibrium taken into account are shown in Fig. 6 (curves 1 and 2 are the eroded rough and smooth nozzles according to two-dimensional gasdynamics, respectively, and 1' is the rough eroded nozzle according to one-dimensional gasdynamics). Roughness increases the loss of specific impulse by friction.

The influence of nonstationarity of the nozzle wall heating process is observed in Figs. 3-6. This influence is especially noticeable for small nozzles (large relative wall thickness) and their short operating times. Not taking account of the nonstationarity and the boundary-layer growth in such situations is especially fraught with great errors. The complex problem presented permits obtaining much useful information on taking account of the influence of diverse factors on the individual loss components of the nozzle specific impulse, the two-phase flow parameters, and the boundary-layer characteristics.

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ONE-DIMENSIONAL PULSATION OF A TOROIDAL GASEOUS CAVITY IN A COMPRESSIBLE LIQUID

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Let us discuss within the framework of the acoustic approximation the problem of the pulsation of a toroidal cavity formed as a result of the explosion of a ring-shaped explosive charge on condition of the fulfillment of the inequality $a \gg R$, where a =const is the radius of the torus and R is the radius of the cavity. At the same time the cross section of the toroidal cavity practically preserves the shape of a true circle, as the experimental data show, during a single pulsation period when $a \approx 10^3 R_*$ and during a single half-period of pulsation when $a \approx 10^2 R_*$. (R_* is the radius of the charge). The problem of the pulsation of a gaseous torus in an incompressible liquid has been discussed in [1]; however, it does not offer the possibility of evaluating such an important parameter as the maximum radius of the expanding cavity, and consequently, the energy distribution among the detonation products and the shock wave in the case of an explosion with axial symmetry.

The solution of the indicated problem is fraught with many difficulties, in particular, the complexity of the solution of the wave equation. Therefore, it is necessary first of all to find a method of constructing an equation of one-dimensional pulsation which would permit simplifying the problem posed. Since an expression for the velocity potential can be found for a number of spatial potential problems of an ideal incompressible liquid in the case of specified assumptions, an attempt to use it for the transition to acoustic models is natural. The practicability of this method is shown below in the example of the construction of the equation of one-dimensional pulsation of bubbles.

§1. Let the velocity potential in the case of an incompressible liquid have the form $\varphi = \Phi(t)/f(r)$. Then its acoustic version can be represented as $\varphi = \Phi(t - r/c_0)/f(r)$. Since potential flow of a liquid $u = -\nabla\varphi$ is being discussed, where u is the velocity of a fluid particle, then

$$u = \Phi f_r/f^2 + \Phi'/c_0 f, \tag{1.1}$$

where the prime denotes a derivative with respect to $\zeta = t - r/c_0$. The Cauchy – Lagrange integral with the form of φ taken into account can be written as

$$\Phi' = f(\omega + u^2/2), \tag{1.2}$$

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